

“Order of Operations” Sometimes, always, or never...

COABE 2016 Texas



**COABE/TALAE NATIONAL
CONFERENCE IN DALLAS**
BIGGER & BETTER IN TEXAS.

INTRODUCTIONS:

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Mankato Area Adult Basic Education

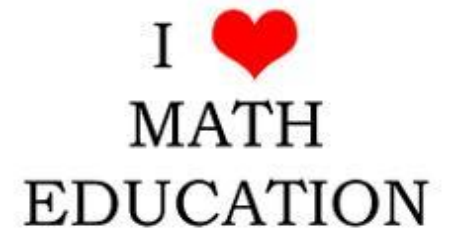
MN Numeracy Initiative, *ABE Teaching Learning Advancement System*
-Advisory team member (since 2011), Project Lead (since 2012)

MN CCRS Implementation team (since 2014)

Online Course Facilitator

World Education (since 2012)

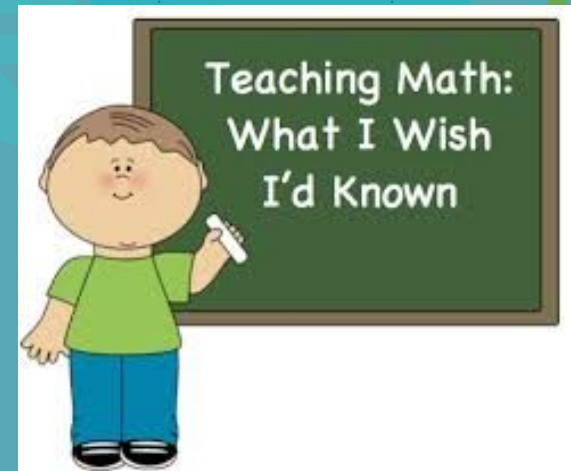
TEAL, *Teaching Excellence in Adult Literacy*, (since 2014)

A white rectangular box containing the text "I ❤️ MATH EDUCATION". The heart is a solid red color. The text is in a black, serif font, with "I" and "MATH" on the top line and "EDUCATION" on the bottom line.

I ❤️
MATH
EDUCATION

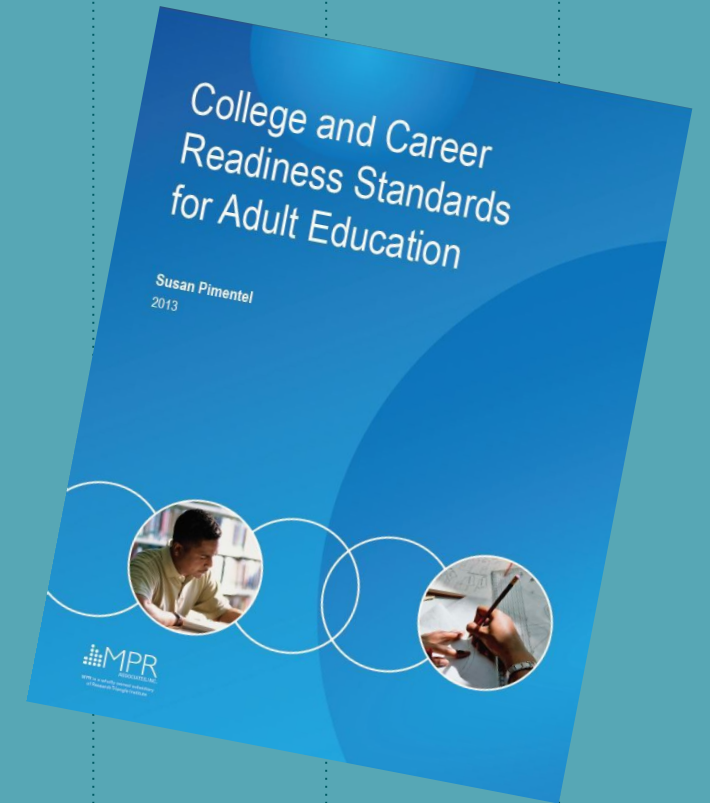
Objectives:

- Review mathematical operations and properties
- Look at “order of operations” through the lens of the mathematical shift, rigor, (specifically the component conceptual understanding)
- Apply understanding to simplifying algebraic expressions
- Use understanding of order of operations to solve Algebraic equations.



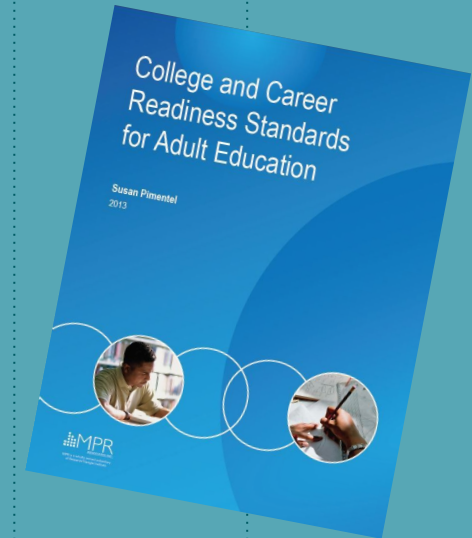
College and Career Readiness Standards

- Specific Mathematical Content Standards
- Standards for Mathematical Practice
- Mathematical Shifts



STANDARDS FOR MATHEMATICAL PRACTICE

- Make sense of problems and persevere in solving them. (MP.1)
- Reason abstractly and quantitatively. (MP.2)
- Construct viable arguments and critique the reasoning of others. (MP.3)
- Model with mathematics. (MP.4)
- Use appropriate tools strategically. (MP.5)
- Attend to precision. (MP.6)
- Look for and make use of structure. (MP.7)
- Look for and express regularity in repeated reasoning. (MP.8)



Mathematical Shifts:

Shift 1 – Focus: Focusing strongly where the standards focus

Shift 2 – Coherence: Designing learning around coherent progressions level to level

**Shift 3 – Rigor: Pursuing conceptual understanding, procedural skill and fluency, and application
–all with equal intensity**

College and Career
Readiness Standards
for Adult Education

Susan Pimentel
2013



MPR
MICHIGAN POSTSECONDARY
RESEARCH

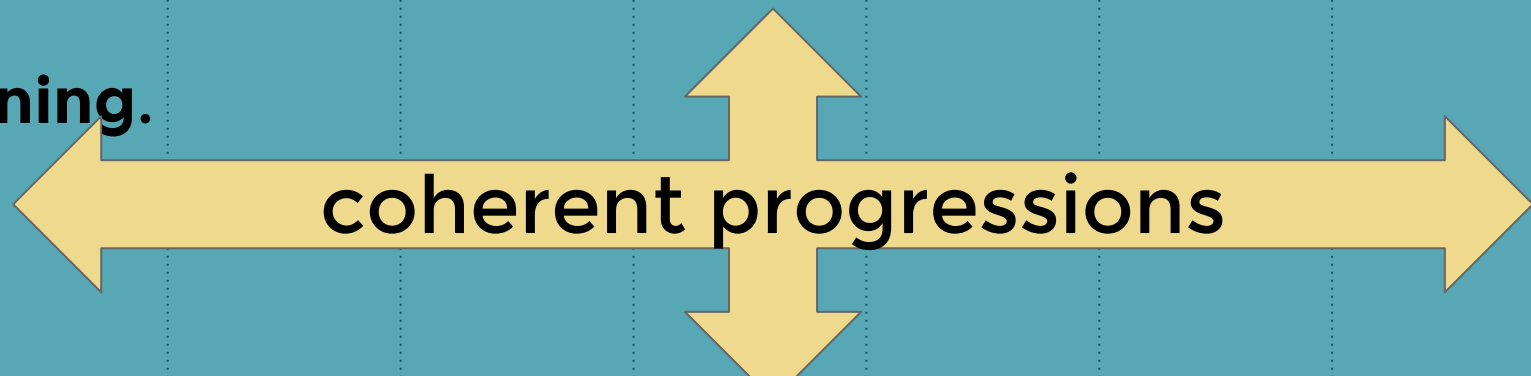
Shift 1 – Focus: Focusing strongly where the standards focus

- **narrow significantly and to**
- **deepen the manner in which they teach mathematics,**
 - **secure the mathematical foundations,**
 - **conceptual understanding,**
 - **procedural skill and fluency, and**
 - **ability to apply the math they have learned .**
outside the math classroom.



Shift 2 – Coherence: Designing learning around coherent progressions level to level

- **create coherent progressions in the content within and across levels,** so that students can build new understanding onto previous foundations. That way, instructors can count on students having **conceptual understanding of core content.** Instead of each standard signaling a new concept or idea, standards at higher levels become **extensions of previous learning.**



What's the answer to this?

$$4 + 10 \div 2 = ?$$

Did you get 7?

Do you see how the answer could be 9?

There are two possible ways to do this problem:

$$4 + 10 \div 2$$

Do the **addition**
first:

$$\begin{array}{r} 4 + 10 \div 2 \\ 14 \div 2 \\ 14 \div 2 \\ 7 \end{array}$$

Do the **division**
first:

$$\begin{array}{r} 4 + 10 \div 2 \\ 4 + 5 \\ 4 + 5 \\ 9 \end{array}$$

CONTINUE ▶

Why Do We Need an Order of Operations?

Page 2 of 2

Which is it?

Both answers can't be right or we'd always be arguing about the answers to math problems. The nice thing about math is that there's always just ONE answer!

So, a long time ago, math geeks decided to make a set of rules for what to do first in a math problem.

These rules are called "the order of operations."

I'll tell you what the rules are in the next lessons...
For now, remember this word:

P**E****M****D****A****S**

Go to the next lesson to find out if the answer to our problem is 7 or 9! And go all the way through to find out what all the letters of **P****E****M****D****A****S** stand for!

Basic Operations:

Addition:

Subtraction:

Multiplication:

Division:



Key ideas for Addition/Subtraction:

Addition:

Subtraction:

$$1286 - 99 =$$

$$\begin{array}{r} 1286 \\ -99 \\ \hline \end{array}$$

OR

$$\begin{array}{r} 1286 \\ -99 \\ \hline \end{array}$$



Key ideas for Multiplication:

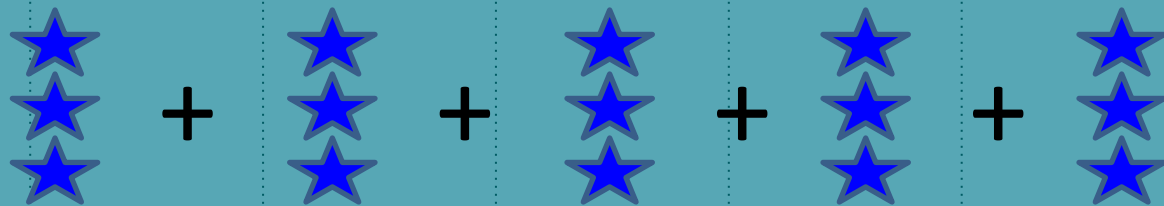
$$\left(\begin{array}{c} \text{Number of} \\ \text{groups} \end{array} \right) \times \left(\begin{array}{c} \text{Number per} \\ \text{group} \end{array} \right) = \begin{array}{c} \text{total number} \\ \text{of items} \end{array}$$

$$\left(\begin{array}{c} \text{Number per} \\ \text{group} \end{array} \right) \times \left(\begin{array}{c} \text{Number of} \\ \text{groups} \end{array} \right) = \begin{array}{c} \text{total number} \\ \text{of items} \end{array}$$

Multiplication

$$(5) \times (3) = 15$$

$$(5) \text{ "Equal groups of" } (3) = 15$$



Key ideas for Division:

Number of items per
group

Number of groups

total number

Number of groups

Number of items per
group

total number

PRIOR/FOUNDATIONAL KNOWLEDGE:

DISCOVER CONCEPTUAL LINKS TO PREVIOUS SKILLS -(COHERENCE)

- What does an “equal sign” mean?
- Math properties:
 - Additive Inverse: $4 + -4 = 0$
 - Additive Identity: $8 + 0 = 8$
 - Multiplicative Inverse: $5 \times \frac{1}{5} = 1$
 - Multiplicative Identity: $7 \times 1 = 7$
 - Commutative property of Multiplication/Addition
 - Associative property of Multiplication/addition
 - Definition: subtraction
 - Definition: division

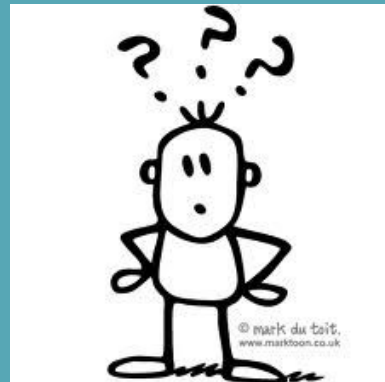


Others..

- Order of operations

WHAT IS ORDER OF OPERATIONS?

PEMDAS	PERMDAS	BEDMAS	BODMAS/ BIDMAS
Parenthesis Exponent Multiply/Divide Add/Subtract	Parenthesis Exponent/Roots Multiply/Divide Add/Subtract	Brackets Exponents Divide/Multiply Add/Subtraction	Brackets Orders (Indices) Divide/Mult Add/Subt



*Please Excuse My Dear
Aunt Sally!!!*





RIGHT WAY
RIGHT WAY
RIGHT WAY
RIGHT WAY

HOW MUCH MONEY?



(Each pile above contains one \$20, one \$10, and one \$5.)



COUNT...



$$\$20 + \$10 + \$5$$



$$\$20 + \$10 + \$5$$



$$\$20 + \$10 + \$5$$



$$\$1$$



$$\$5 + \$5$$

WRITE AS AN EXPRESSION...



\$1



\$5

\$5



$\$20 + \$10 + \$5$

$\$20 + \$10 + \$5$

$\$20 + \$10 + \$5$

WRITE AS AN EXPRESSION...

\$1

\$5

$\$20 + \$10 + \$5$

\$5

$\$20 + \$10 + \$5$

$\$20 + \$10 + \$5$

One dollar

2 “fives”

Three groups:
twenty, ten & five

1

+

2(5)

+

3 (20+10+5)

What is the most natural step to do first?

USING ORDER OF OPERATIONS...

$$1 + 2(5) + 3(20+10+5)$$

-Simplify within the parentheses

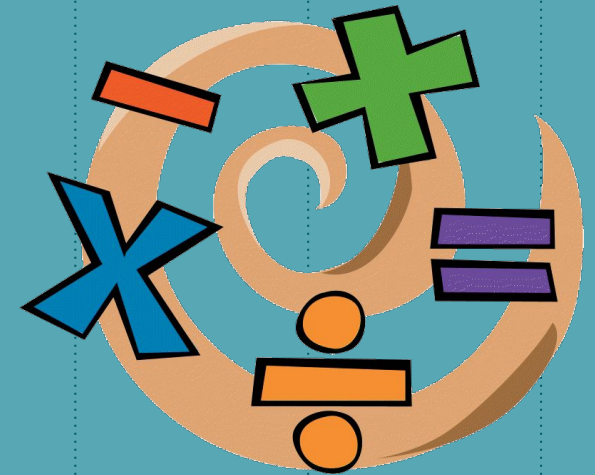
$$1 + 2(5) + 3(35)$$

-Multiply/Divide

$$1 + 10 + 105$$

-Add/Subtract

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NOT ORDER OF OPERATIONS...

$$1 + 2(5) + 3(20+10+5)$$

-Multiply using distributive property.

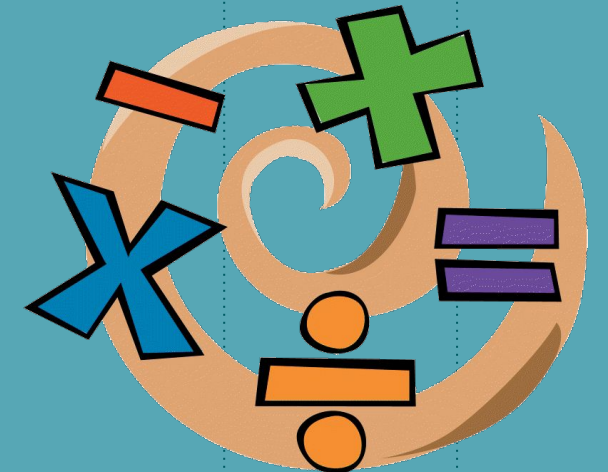
$$1 + 2(5) + 3(20) + 3(10) + 3(5)$$

-Multiply/Divide

$$1 + 10 + 60 + 30 + 15$$

-Add/Subtract

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Thoughts?



HOW DOES THIS HELP US IN SOLVING ALGEBRAIC EQUATIONS?

Key Shift: Coherence

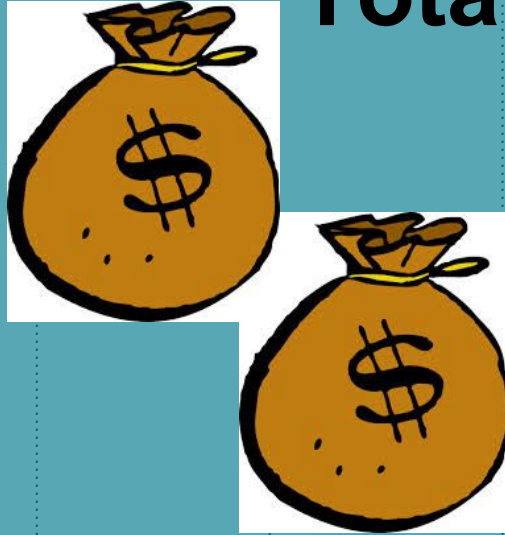
Creating connections across the content areas, so students can build new understanding onto previous foundations.

Using the order of Operations, let's solve the algebraic problem by "retracing our steps..."

Total \$126



\$1



\$?

\$?



\$20+\$10+\$5

\$20+\$10+\$5

\$20+\$10+\$5

$$1+2(x)+3(20+10+5)=126$$

$$1+2(x)+3(20+10+5)=126$$

1. Simplify- using order of operations...
2. To solve algebraically, - “retrace your steps”, Undo the last step first...

$$1+2(x)+3(20+10+5)=126$$

1. *Simplify- using order of operations...*

$$1+2(x)+3(35)=126$$

Simplify within parentheses

$$1+2x+70=126$$

Simplify, adding like terms

$$2x+71=126$$

What's happening?

*Use the inverse to “undo” it...
(Retracing our steps...)*

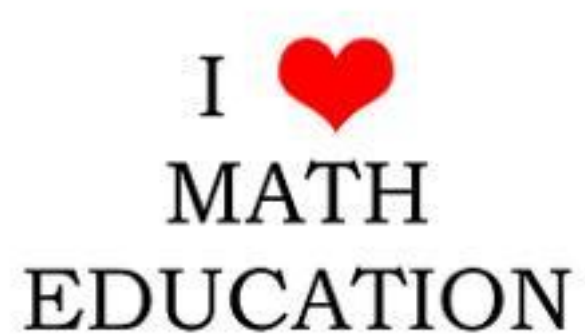
Rather than an unbreakable rule- “Order of Operations”

Sometimes....

Always....

Never...





Thank
You!

